1212-16-505-001

Code No. 3363 / CORE

FACULTY OF SCIENCE

M.Sc. IV – Semester Examination, May / June 2018

Subject: Mathematics

Paper – II

General Measure Theory

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Define a measure μ on a measurable space (x, β). Prove that μ is countably sub additive.

2 State and prove Monotone convergence theorem.

3 Prove that countable union of positive sets ia a positive set.

4 Suppose (x, β, μ) is a measure space and f is an integrable function on X w.r.t. μ. Prove that ν defined on β by $\nu(E) = \int_E f d\mu$ is a signed measure on β.

5 Define a μ^*-measurable set E. Suppose. $\mu^*(E) = 0$, prove that E is a μ^*-measurable set.

6 Suppose $E \subset x$ xy and x ϵ x. Define x – cross section of E with usual notations. Prove that

i) $\psi_{E_x}(y) = \psi_E(x, y) \; \forall \; y \in y$

ii) $\bar{E}_x = (\bar{E})_x$

7 Suppose μ^* and μ_* are the outer and inner measures induced by a measure μ on an algebra A of subsets of X. Prove that $\mu_*(E) \le \mu^*(E) \; \forall \; E \in P(x)$.

8 If A ϵ A prove that

$\mu(A) = \mu_*(A \cap E) + \mu^*(A \cap \bar{E})$.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Suppose (x, β, μ) is a measure space. Prove that is can be extended to a complete measure space (x_1, β_0, μ_0) where $\beta \subset \beta_0$ and restriction μ_0 to β is μ i.e. $\mu_0 / \beta = \mu$.

OR

b) Suppose (x, β) is a measurable space and $E \in \beta$. Suppose $f: E \to [-\infty, \infty]$ is a mapping. Prove that the following are equivalent.

i) $\{x \in E: f(x) > \alpha\} \in \beta$ for all $\alpha \in R$

ii) $\{x \in E: f(x) \geq \alpha\} \in \beta$ for all $\alpha \in R$

iii) $\{x \in E: f(x) < \alpha\} \in \beta$ for all $\alpha \in R$

iv) $\{x \in E: f(x) \leq \alpha\} \in \beta$ for all $\alpha \in R$

10 a) Suppose E is a measurable set such that $0 < \nu(E) < \infty$. Prove that E has a positive set A such that $\nu < A_1 > 0$.

OR

b) State and prove Jordon – decomposition theorem.

11 a) Prove that the class β of all μ^* - measurable sets is a σ - algebra of sets.

OR

b) Suppose (x, A, μ) and (y, β, ν) are complete measure spaces and R is the class of all measurable rectangles in X x Y. Suppose $E \in R_{\sigma\delta}$ with $(\mu \times \nu)(E) < \infty$.

Prove that

i) The function $g: x \to [0, \infty]$ defined by $g(x) = \nu(E_x)$ $\forall x \in X$ is a measurable function on x. Also

ii) $\int_X g\, d\mu = \int_X \nu(E_x) d\mu = (\mu \times \nu)(E)$.

12 a) Suppose $E \subset x$ with $\mu^*(E) < \infty$. Prove that E is μ^* - measurable if and only if

$\mu^*(E) = \mu_*(E)$.

OR

b) State and prove Coratheodory outer measure theorem.

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