1212-16-505-001

Code No. 3361 / CORE

FACULTY OF SCIENCE

M.Sc. IV-Semester Examination, May / June 2018

Subject : MATHEMATICS

Paper - I

Advanced Complex Analysis

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)

1 If $z_1, z_2, \ldots, z_n$ are the zeros of f inside the disc D_R than prove that
$$\int_0^R n(r)\frac{dr}{r} = \sum_{k=1}^{N}\log\left|\frac{R}{z_k}\right|$$

2 Find the growth order of sin πz.

3 Prove that the Gamma function extends to an analytic function in the half plane Re(s) > 0.

4 For n ∈ IN, prove that $\mathrm{Res}_{s=-n}\,\Gamma(s) = \frac{(-1)^n}{n!}$.

5 Prove that $(\zeta(s))^2 = \sum_{n=1}^{\infty}\frac{d(n)}{n^s}$

6 If Re(s) >l, prove that $\log \zeta(s) = \sum_{p,m}\frac{p^{-ms}}{m}$, where p is prime, m ∈ IN.

7 For M ∈ SL_2(IR), prove that f_M maps H onto itself where H is the upper half plane.

8 Prove that $\psi_\alpha(z) = \frac{\alpha - z}{1-\bar{\alpha}z}$, where $\alpha \in \mathbb{C}$, $|\alpha|<1$ is an automorphism of the unit disc D.

PART – B (4 x 12 = 48 Marks)

9 a) Find the Hadomards products for
i) $e^z - 1$ ii) sin πz

OR

b) State and prove Jensen's formula.

10 a) Prove that $\lim_{n\to\infty}\frac{n^s\, n!}{s(s+1).....(s+n)} = \Gamma(s)$ for $s \neq 0, -1, -2, \ldots$

OR

b) Prove that $\Gamma(s)\Gamma\left(s+\frac{1}{2}\right) = \sqrt{\pi}\; 2^{1-2s}\,\Gamma(2s)$.

11 a) Prove that, if $\psi_1 \sim \dfrac{x^2}{x}$ as $x \to \infty$, then prove that $\psi(x) \sim x$ as $x \to \infty$.

OR

b) Show that the function $\xi(s) = \pi^{-\frac{s}{2}}\,\Gamma\left(\dfrac{s}{2}\right)\xi(s)$ is real when s is real or when $\text{Re}(s) = \dfrac{1}{2}$.

12 a) State and prove Schwarz's lemma.

OR

b) Prove that every antomorphism of upper half plane H takes the form f_M for some $M \in SL_2$ (IR).

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