Code No. 2451 / Core

FACULTY OF SCIENCE

M.Sc. III – Semester Examination, January 2018

Subject: Mathematics

Paper – III (A)
Discrete Mathematics

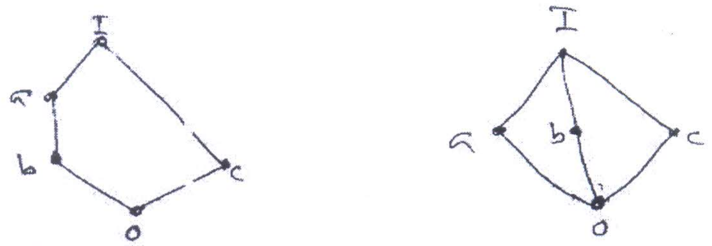
Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Let L be a lattice. Then for every a and b in L,
a) $a \vee b = b$ if and only if $a \leq b$,
b) $a \wedge b = a$ if and only if $a \leq b$,
c) $a \wedge b = a$ if and only if $a \vee b = b$.

2 Show that the lattices pictured in following figure are non-distributive.



(a) (b)

3 Show that if n is a positive integer and p^2/n, where p is a prime number, then prove that D_n is not a Boolean algebra.

4 Show that in a Boolean algebra, for any a and b,

$(a \wedge b) \vee (a \wedge b') = a$.

5 If a graph G has more than two vertices of odd degree, then prove that there can be no Euler path in G.

6 Let the number of edges of G be m. Then prove that G has a Hamiltonian circuit if $m \geq \frac{1}{2}(n^2 - 3n + 6)$ (here n is the number of vertices).

7 Let (T, v_0) be a rooted tree on a set A. Then prove that
a) T is irreflexive
b) T is asymmetric
c) If $(a,b) \in T$ and $(b, c) \in T$, then $(a, c) \notin T$, for all a, b, and c in A.

8 Prove that a tree with n vertices has n – 1 edges.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) If $s_1 = \{x_1, x_2, \ldots x_n\}$ and $s_2 = \{y_1, y_2, \ldots y_n\}$ are any two finite sets with n elements, then prove that the lattices $(p(s_1), \subseteq)$ and $(p(s_2), \subseteq)$ are isomorphic.

OR

b) Let L be a lattice. Then L holds the following:

1) Idempotent properties
2) Commutative properties
3) Associative properties
4) Absorption properties.

10 a) Show that in a Boolean algebra for any a, b, and c.

$(a \wedge b \wedge c) \vee (b \wedge c) = b \wedge c$.

OR

b) Let $n = p_1 p_2 \ldots p_k$, where the p_i are distinct primes. Then prove that D_n is a Boolean algebra.

11 a) What is the total number of edges in K_n, the complete graph on n vertices? Justify your answer.

OR

b) Draw the complete graph on seven vertices.

12 a) Let (T, v_0) be a rooted tree. Then prove the following:

i) There are no cycles in T
ii) v_0 is the only root of T
iii) Each vertex in T, other than v_0, has in-degree one, and v_0 has in-degree zero.

OR

b) If (T, v_0) is a rooted tree and $v \in T$, then prove that $T(v)$ is also a rooted tree with root v.

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