310 1212-16-505-001

Code No. 2447 / Core

FACULTY OF SCIENCE

M.Sc. III – Semester Examination, January 2018

Subject: Mathematics

Paper – I
Complex Analysis

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Prove that if a set contains each of its accumulation points, then it must be a closed set.

2 Show that $|\sin z|^2 = \sin^2 x + \sin h^2 y$.

3 Let f(z) = 1 or 4y according as y < 0 or y > 0 and C be the arc from z = -1 – i to z = 1 + i along the curve $y = x^3$. Compute $\int_C f(z)dz$.

4 State and prove fundamental theorem of algebra.

5 Find Taylor series of cos z about the point $z = \frac{\pi}{2}$.

6 Evaluate $\int_{|z|=2} \frac{5z-2}{z(z-1)} dz$, where |z| = 2 is positively oriented.

7 Compute $\int_0^\infty \frac{\sin x}{x} dx$.

8 Find the image of the region x > 1, y > 0 under the transformation $w = \frac{1}{2}$.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) i) State and prove the reflection principle.
ii) Solve sin h z = i.

OR

b) i) Derive Cauchy-Riemann equations for a function $f(z) = x + i\nu$ which is differentiable at a point z_0.

ii) Suppose that f(z) and $\overline{f(z)}$ are both analytic in a domain D. Show that f(z) is a constant.

10 a) State and prove Cauchy-Goussat theorem.

OR

b) State and prove Cauchy's integral formula.

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11 a) i) Derive a formula for the residue at ∞ for a function f(z).

ii) Evaluate $\displaystyle\int_{|z|=3} \frac{e^{-z}}{(z-1)^2}\,dz.$

OR

~~b) i)~~ Compute $\displaystyle\int_{|z|=2} \frac{dz}{z^3(z+4)}$

~~ii) Evaluate~~ $\displaystyle\int_{|z|=2} \frac{\cosh \pi z}{z(z^2+1)}\,dz.$

12 a) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{x \sin x \, dx}{(x^2+1)(x^2+4)}.$

OR

~~b)~~ State and prove Rouche's theorem.

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