Code No. 3002/CORE

FACULTY OF SCIENCE
M.Sc. II – Semester Examination, May / June 2018

Subject: Mathematics

Paper – IV : Topology

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Let X be an infinite set. Show that the empty set ϕ together with all subsets of X whose complements are finite is a topology on X.

2 Let X be a topological space and A an arbitrary subset of X. Then prove that $\bar{A}$={x : each neighbourhood of x intersects A}

3 Show that any continuous image of a compact space is compact.

4 Prove that totally bounded metric space is bounded.

5 Show that a closed subspace of a normal space is normal.

6 Show that a topological space is a T_1 – Space if and only if each point is a closed set.

7 Define component of a topological space x, and prove that each point in X is contained in exactly one component of X.

8 Let X be a topological space. If $\{A_i\}$ is a non-empty class of connected subspaces of X such that $\bigcap_i A_i$ is non-empty, then prove that $A = \bigcup_i A_i$ is also a connected subspace of X.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) Let X be a non-empty set and let there be given a class of subsets of X which is closed under the formation of arbitrary intersections and finite unions. Then show that the class of all complements of these sets is a topology on X whose closed sets are precisely those initially given.

OR

(b) Let X be any non-empty set, and let S be an arbitrary class of subsets of X. Then prove that S can serve as an open subbace for a topology on X, in the sense that the class of all unions of finite intersections of sets in S is a topology.

10 (a) State and prove Ascoli's theorem.

OR

(b) (i) Show that a metric space is sequentially compact ⇔ it has the Bolzano Weierstrass property.

(ii) Show that a closed subspace of a complete metric space is compact ⇔ it is totally bounded.

11 (a) State and prove Urysohn's imbedding theorem.

OR

(b) State and prove Tietze's extension theorem.

12 (a) Show that a subspace of real line **R** is connected if and only if it is an interval. Hence prove that **R** is connected.

OR

(b) (i) Let X be a compact Hausdorff space. Then prove that X is totally disconnected ⇔ it has an open base whose sets are also closed.

(ii) Let X be a locally connected space. Prove that if Y is an open subspace of X, then each component of Y is open in X. In particular, each component of X is open.

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