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Code No. 2998/CORE

FACULTY OF SCIENCE
M.Sc. II – Semester Examination, May / June 2018

Subject: Mathematics

Paper – III : Theory of Ordinary Differential Equations

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Prove that x^4 and $|x|x^3$ are linearly independent functions on [-1. 1] but they are linearly dependent on [-1, 0] and [0, 1].

2 Prove that there are three linearly independent solution of the third order equation $x''' + b_1(t)x'' + b_2(t)x_1 + b_3(t)x = 0$, $t \in I$ on an interval I.

3 Let h = min $\{a, \frac{b}{L}\}$ then the successive approximations given by

$$x_n(t) = x_0 + \int_{t_0}^{t} f(s, x_{n-1}(s))ds, \quad n = 1, 2, 3\ldots$$ are valid on $I = |t - t_0| \le h$. Further

$|x_j(t) - x_0| \le L\,|t - t_0| \le b$, j = 1, 2, ..., $t \in I$.

4 Prove the error $x(t) - x_n(t)$ satisfies the estimate $|x(t) - x_n(t)| \le \frac{L(Kh)^{n+1}}{K(n+1)} e^{kh}$,

$t \in [t_0, t_0+h]$.

5 Suppose that f(t, x) is non increasing in x then prove that there exist lower and upper solutions v_0, ω_0, of $x' = f(t, x)$, $x(t_0) = x_0$ such that $v_0 \le \omega_0$ on $I = [t_0, t_0 + h]$

6 State Ascoli's lemma.

7 The equation $L_2(y) = a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$ is self adjoint if and only if $a_0^1(x) = a_1(x)$.

8 If $y_1(x) = x$ is one solution of
$x^3y'' - xy' + y = 0$, $x > 0$ then find second solution $y_2(x)$.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) Let $b_1, b_2, \ldots, b_n$ be real or complex valued functions defined and continuous on an interval I and $\phi_1, \phi_2, \ldots, \phi_n$ are n solutions of the equation $L(x)(t) = x^{(n)}(t) + b_1x^{(n-1)}(t) + \ldots + b_n(t)x(t) = 0$ existing on I then show that they are linearly independent on I if and only $w(t) \neq 0$ for every $t \in I$.

OR

(b) Solve $x'' - 4x' = te^{4t}$ by the method of undetermined coefficients.

10 (a) Let $x(t) = x(t, t_0, x_0)$ and $x^*(t) = x(t, t_0^*, x_0^*)$ be solutions of the IVPs $x' = f(t, x)$, $x(t_0) = x_0$ and $x' = f(t, x)$, $x(t_0^*) = x_0^*$ respectively on an interval $a \le t \le b$. Let $f \in Lip(D, K)$ be bounded by L in D then show that for any $\in > 0$, there exists $\delta = \delta(\in) > 0$ such that $|x(t) - x^*(t)| < \in$, $a \le t \le b$ where $|t_0 - t_0^*| < \delta$ and $|x_0 - x_0^*| < \delta$

OR

(b) Prove that the IVP $x' = f(t, x)$, $x(t_o) = x_o$ has a unique solution defined on $t_o \le t \le t_o + h$, $h > 0$ if the function $f(t, x)$ is continuous in the strip $t_o \le t \le t_o + h$, $|x| < \infty$ and satisfies the Lipschitz condition.
$|f(t, x_1) - f(t, x_2)| \le K\,|x_1 - x_2|$, $K > 0$ K being Lipschitz constant

11 (a) Let V, W $\in C'\{[t_o, t_o + h), R\}$ be lower and upper solutions of $x' = f(t, x)$, $x(t_o) = x_o$ respectively. Suppose that for $x \ge y$, f satisfies the equality $f(t, x) - f(t, y) \le L(x - y)$. Where L is a positive constant then prove that $v(t_o) \le w(t_o)$ implies that $v(t) \le w(t)$, $t \in [t_o, t_o h]$.

OR

(b) Let $f, v \in C[R^+, R^+]$, $w \in C(R^{+2}, R^+)$ and $C > 0$ satisfy

$f(t) \le C + \int_{t_o}^{t} v(s) f(s) + w(s, f(s)] ds, t \ge t_o$ suppose, further that w(t, z exp

$(\int_{t_o}^{t} v(s)ds)) \le \lambda(t) g(z) \exp(\int_{t_o}^{t} v(s)ds)$ where $\lambda \in C[R^+, R^+]$, $g \in [(0, \infty), (0, \infty)]$ and

g(u) is non decreasing in u then prove that $f(t) \le G^{-1}[G(C) +$

$\int_{t_o}^{t} \lambda(s)ds] \exp(\int_{t_o}^{t} v(s)ds)$, $t_o \le t \le T$. Where $G(u) - G(u_o) = \int_{u_o}^{u} \frac{ds}{g(s)}$, $G^{-1}(u)$ is the

inverse function of G(u) and T = sup [$t \ge t_o$ / G(C) + $\int_{t_o}^{t} v(s)ds \in \mathrm{dom}G^{-1}\}$

12 (a) State and prove Abel's formula.

OR

(b) State and prove Bocher-Osgood theorem.

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