Code No. 2996 / CORE

FACULTY OF SCIENCE

M.Sc. II – Semester Examination, May / June 2018

Subject: Maths / Applied Maths

Paper – II
Advanced Analysis

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Define an algebra and σ – algebra of sub sets of X. Give an example of an algebra of sets which is not a σ – algebra of sets.

2 Define a measurable function on a measurable set. Prove that every constant function defined on a measurable set is a measurable function.

3 Suppose f is a bounded function defined on [a,b]. Prove that f is measurable if f is Riemann integrable on [a,b]. Also prove that it's Lebesque and Riemann integrals are equal.

5 State and prove Lebesque's dominated convergence theorem.

6 Suppose f, g are functions of bounded variation on [a,b]. Prove that f + g is also a function of bounded variation on [a,b]. Also prove that $T_a^b(f+g) \le T_a^b(f) + T_a^b(g)$.

7 Suppose f is integrable on [a,b] and F(x) = $\int_a^x f(t)dt \ \forall x \in [a,b]$. If $F(x) = 0 \ \forall x \in [a,b]$. Prove that f = 0 a.e.

8 Prove that L^{∞} [0, 1] is a normed linear space.

(8) S.T. the greatest integer function on $f(x) = [x]$ is not a continuous but a function of b.v. on $[0, 3]$.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) i) If E_1 and E_2 are measurable. Prove that $E_1 \cup E_2$ is also measurable.

ii) Suppose $A \subseteq R$ and $E_1, E_2, \ldots E_n$= are finite collection of pair-wise disjoint measurable sets in R. Prove that $m^*\left(A \cap \left(\bigcup_{i=1}^{n} E_i\right)\right) = \sum_{i=1}^{n} m^*\left(A \cap E_i\right)$.

OR

b) Prove that there exists a non-measurable subset of R.

10 a) Suppose ϕ and ψ are simple functions which vanishes outside a set of finite measure. Suppose the measure of the set on which both ϕ and ψ vanishes simultaneously is finite. Prove that

i) $(a\varphi + b\psi) = a\int\varphi + b\int\psi$

ii) if $\varphi \geq \psi$ then $\int\varphi \geq \int\psi$

OR

b) Suppose f is a non-negative measurable function which is integrable over a measurable set E. Prove that given any $\epsilon > 0$ there exists a $\delta > 0$ such that for all subsets $A \subset E$ with $m(a) < \delta$ wehave $\int_A f < \epsilon$.

11 a) Suppose $\{f_n\}$ is a sequence of measurable functions which converges in measure to a limit function f on a measurable set E. Prove that there exists a subsequence $\{f_{n_k}\}$ of $\{f_n\}$ such that $f_{n_k} \to$ f a.e. on E.

OR

b) Suppose f is a real valued function defined on [a,b] and P is a portion of [a, b]. Define positive variation sum P, negative variation sum n, total variation sum t of f w.r.t. the portion P. Also define positive variation P, negative variation N, and total variation T of f on [a, b]. Prove that

i) $p - n = f(b) - f(0)$

ii) $p + n = t$

iii) $P - N = f(b) - f(0)$

iv) $P+N = T$.

12 a) Prove that a real valued function F defined on [a, b] is an indefinite integral if and only if F is absolutely continuous on [a, b].

OR

b) Prove that a normed linear space X is complete if and only if every absolutely summable series in X is summable in X.

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partial