Code No. 2061 / CORE

FACULTY OF SCIENCE

M.Sc. I-Semester Examinations, January 2018

Subject : Maths/ Applied Maths/ Maths with Computer Science

Paper – II : Analysis

Time: 3 hours **Max. Marks: 80**

PART-A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Define metric space. Give an example.

2 Prove that every neighborhood is an open set in any metric space.

3 Prove that a continuous real valued function defined on a compact metric space is bounded and attains its bounds.

4 Let f and g be complex continuous functions on a metric space X. Then prove that f + g and fg are continuous.

5 Prove that $\underline{\int_a^b} f\, d\alpha \leq \overline{\int_a^b} f\, d\alpha$.

6 If f is a monotonic on $[a,b]$ and if α is continuous on $[a,b]$, then prove that $f \in R(\alpha)$.

7 If $\{f_n\}$ is a sequence of continuous functions on E, and if $f_n \to f$ uniformly on E, then prove that f is continuous on E.

8 Examine the uniform convergence of the sequence $\{f_n\}$ defined by

$$f_n(x) = \frac{1}{1+n^2 x}, \quad 0 < x < 1, \ n=1, 2, \ldots$$

PART- B (4 x 12 = 48 Marks)
(Essay Answer type)

9 a) If a set E in R^k has one of the three following properties then prove that it has the other two

(i) E is closed and bounded.

(ii) E is compact,

(iii) Every infinite subset of E has a limit point in E.

OR

b) Prove that every K - cell is compact.

10 a) (i) A mapping f of a metric space X into a metric space Y is continuous on X if only if $f^{-1}(G)$ is open in X for every open set G in Y.

(ii) Suppose f is a continuous mapping of a compact metric space X into a metric space Y, then Prove that f(X) is compact.

OR

b) (i) Let f be a continuous mapping of a compact metric space X into a metric space Y, then prove that f is uniformly continuous on X.

(ii) Show that a mapping f of a metric space Y is continuous if and only if $f^{-1}(c)$ is closed in X for every closed set c in y.

11 a) Prove that $f \in R(\alpha)$ on [a, b] if and only if for every $\in > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

OR

b) Suppose $f \in R(\alpha)$ on [a, b] $m \leq f \leq M$, Φ is continuous on [m, M], and $h(x) = \Phi(f(x))$. Then prove that $h \in R(\alpha)$ on [a, b].

12 a) If f is a continuous complex function on [a b] then prove that there exists a sequence of polynomials $\{P_n\}$ such that $\lim_{n\to\infty} P_n(x) = f(x)$ uniformly on [a, b].(If f is real, then $\{P_n\}$ may be taken real)

OR

b) Prove that there exists a real continuous function on the real line which is no where differentiable.

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