Code No. 7078

FACULTY OF SCIENCE

M.Sc. IV-Semester Examination, May / June 2017

Subject : Mathematics/Applied Maths

Paper - V (a)

Calculus of Variations

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Define a linear functional and variational functional. Give one example each.

2 On what curves can the functional

$$V[y(x)] = \int_0^{\pi/2} \left(y'^2 - y^2\right) dx;\ y(0) = 0,\ y(\pi/2) = 11$$ be extremized.

3 Define Brachisto chrone problem.

4 Find the extremals of the functional

$$V[y(x), z(x)] = \int_0^{\pi/2} \left(y'^2 - z'^2 + 2yz\right) dx.$$

5 Find the extremals of the functional

$$V[y(x), z(x)] = \int_{x_0}^{x_1} \left(16y^2 - y''^2 + x^2\right) dx.$$

6 Find the Euler Ostrogradsky equation for the functional

$$S[z(x,y)] = \iint \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\, dy$$

7 Derive the equations of motion of a projectite in space using Hamiltons equation.

8 Derive the differential equation of motion of simple pendulum using Lagrange's equation.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 a) i) State and prove the fundamental Lemma of calculus of variations.

ii) Prove that the shortest distance between two points in a place is a straight line.

OR

b) Derive Euler's equation for the functionals of the form

$$V[y(x)] = \int_{x_0}^{x_1} F(x, y, y')\, dx, \quad y(x_0) = y_0, \quad y(x_1) = y_1.$$

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10 a) Define minimum surface of revolution problem and show that it is a family of catenaries.

OR

b) Find the extremals of the functional

$$V[y(x), z(x)] = \int_0^{\pi/2} \left[y'^2 + z'^2 + 2yz\right] dx, \; y(0) = 0, \; y(\pi/2) = 1, \; z(0) = 1, \; z(\pi/2) = -1.$$

11 a) State and derive isometric problem.

OR

b) Derive the Euler-Ostrogradsky equation for the functional

$$v[z(x,y)] = \iint_D F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) dx\, dy.$$

12 a) Derive the differential equation of the free vibrations of a string using the variational principle.

OR

b) Derive the Euler-Poisson equation.

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