Code No. 7076

FACULTY OF SCIENCE

M.Sc. IV-Semester Examination, May / June 2017

Subject : Mathematics

Paper - IV (a)
Banach Algebra

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 In a normed algebra A, prove that (i) the multiplication (x, y) → xy is jointly continuous in its factors and (ii) the product of two Cauchy sequences is Cauchy.

2 Let A be a Banach algebra with unity element. If $z \in A$ and $\|z\| < 1$, then prove that $1 - z$ is invertible.

3 Let A be a Gelfaud algebra. If M is a maximal ideal of A, then prove that M is closed.

4 Prove that every TDZ is singular.

5 If A is a C*-algebra with unit and if $x \in A$ is self-adjoint, then prove that $\sigma_A(x) \subset R$.

6 In a C*-algebra with unity, if a is an element such that $-a^* a \geq 0$ then prove that a = 0.

7 If A is a C*-algebra with unity and if $a \in A$ is normal, then prove that $\sigma(f(a))=f(\sigma(a))$ for all $f \in \mathcal{C}(\sigma(a))$.

8 If $T \in L(H)$, $\|T\| \leq 1$ and f is any complex polynomial, then prove that $\|f(T)\| \leq \|f\|_{\Delta_1}$ where $\Delta_1 = \{\lambda \in \mathbb{C} : |\lambda| \leq 1\}$.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 a) Let A be a Banach algebra with unity. Let U be the set of all invertible elements of A. The prove the following :

i) The mapping $x \to x^{-1}$ $(x \in U)$ is bicontinuous.

ii) If h is a nonzero complex variable, then the $\lim\limits_{h \to 0} \dfrac{(x+h_1)^{-1} - x^{-1}}{h}$ exists and is equal to $-x^{-2}$.

OR

b) i) Let A be a Banach algebra with unity element 1. Prove that for each $x \in A$, $\rho(x)$ is a proper subset of $\mathbb{C}$.

ii) State and prove Gelfaud-Mazur theorem.

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10 a) State and prove Gel'faud representation theorem.

OR

b) If E is a Banach space and $T \in L(E)$, then prove that the following conditions on T are equivalent.
i) T is surjective ; ii) T' is bounded below.

11 a) Prove that a C^*-algebra without unit may be embedded in a C^*-algebra with unity.

OR

b) If A is a C^*-algebra with unity and if f is a state on A, then prove that there exist a unital $*$-representation $\phi : A \to L(H)$ and a vector $u \in H$ such that $f(a) = (\phi(a)u/u)$ for all $a \in A$.

12 a) State and prove Gel'faud-Naimark representation theorem.

OR

b) If $\|T\| \leq 1$ and $f \in \mathbb{C}(t ; \Delta_1)$, then prove that there exists a sequence of complex polynomials f_n such that $\|f_n(T) - f(T)\| \to 0$.

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