Code No. 7071

FACULTY OF SCIENCE

M.Sc. IV-Semester Examination, May / June 2017

Subject : Mathematics

Paper - I

Advanced Complex Analysis

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)

(Short Answer Type)

1 Find the number of roots of $z^4 - 6z + 3 = 0$ in $|z|<1$.

2 Find the residue of cosecz at $z = n\pi$, n an integer.

3 Establish the mean value property of harmonic functions.

4 If f(z) is analytic in $|z|\le 1$ and satisfied $|f| = 1$ on $|z|=1$, show that f(z) is a rational function.

5 Show that $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)=\frac{1}{2}$.

6 Prove that every meromorphic function in the whole plane is the quotient of two entire functions.

7 Prove that $\zeta(s)\Gamma(s) = \int_0^\infty \frac{x^{s-1}}{e^x-1}dx$ Where $\sigma = \text{Re } s > 1$.

8 Prove that residue of $\zeta(s)$ at $s = 1$ is 1.

PART – B (4 x 12 = 48 Marks)

(Essay Answer Type)

9 a) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{a+\sin^2 x}, |a|>1$.

OR

b) State and prove the argument principle.

10 a) State and prove Schwarz's theorem.

OR

b) Establish the Schwarz's formula.

11 a) Derive the Legendre duplication formula.

OR

b) i) Show that $\Gamma\left(\frac{1}{6}\right)=2\left(\frac{3}{\pi}\right)^{\frac{1}{2}}\Gamma\left(\frac{1}{3}\right)^2$

ii) Find the residue of $\Gamma(z)$ at the pole $z = -n$, n a positive integer.

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12 a) For $\sigma = \text{Re}\, s > 1$, prove that

$$\varsigma(s) = -\frac{\Gamma(1-s)}{2\pi i}\int_C \frac{(-z)^{s-1}}{e^z - 1}\,dz,$$ where C is an appropriate infinite path, and $(-z)^{s-1}$ is defined on the complement of the positive real axis as $e^{(s-1)\log(-z)}$ with $-\pi < \text{Im} \log(-z) < \pi$.

OR

b) Derive Jensen's formula.

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