Code No. 6805/O

FACULTY OF SCIENCE

M. Sc. II – Semester Examination, May / June 2017

Subject : Maths / Applied Maths

Paper – I : Advanced Algebra

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 State and prove a Eisenstein criteria.
2 Show that $x^2 - 2$ is irreducible over Q.
3 If f(x) is an irreducible polynomial over F then prove that f(x) has a multiple root if and only if f'(x) =0.
4 Let p be a prime. Then prove that $f(x) = x^p - 1 \in Q(x)$ has splitting field $Q(\alpha)$ where $\alpha \neq 1$ and $\alpha^p = 1$.
5 If E is a finite separable extension of a field F and $H \leq G\left(\frac{E}{F}\right)$ then prove that

$$G\left(\frac{E}{E_H}\right) = H \text{ and } [E:E_H] = \left|G\left(\frac{E}{E_H}\right)\right|.$$

6 Show that the Galois group of $x^4+1 \in Q(x)$ is the Klein's four group.
7 Show that $\phi_8(x)$ and x^8-1 have the same Galois group.
8 If a > 0 is constructible then prove that $\sqrt{a}$ is construction.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) Let E be an extension field of f and u∈E be algebraic over F. If p(x) ∈F(x) is a polynomial of least degree such that q(u)=0 then prove that
(i) p(x) is irreducible over F.
(ii) If g(x) ∈ F(x) is such that u(u)=0 then p(x) / g(x).
(iii) There is exactly one monic polynomial p(x) ∈ F(x) of least degree such that p(u)=0.

OR

(b) Let E be an algebraic extension of a field F and $\sigma : F \rightarrow L$ be an embedding of F into an algebraically closed field L. Then prove that σ can be extended to an embedding $\eta : E \rightarrow L$.

10 (a) (i) Prove that the prime field of a field F is either isomorphic to Q or to $\frac{z}{(p)}$ where p is a prime.
(ii) If the multiplicative group F* of non zero element of a field F is cyclic then prove that F is finite.

OR

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(b) If E is a finite extension of a field F then prove that the following are equivalent.
 (i) $E = F(\alpha)$ for some $\alpha \in E$
 (ii) There are only a finite number of intermediate fields between F and E.

11 (a) State and prove fundamental theorem of Galois theory.

OR

(b) State and prove fundamental theorem of algebra.

12 (a) Let E be a finite extension of F and $G\left(\frac{E}{F}\right)$ be a cyclic group of order n generated by σ. If $w \in E$ is such that $w\,\sigma(w)\sigma^2(w)\ldots\sigma^{n-1}(w) = 1$ then prove that there exists $\alpha \in E^*$ such that $w = \sigma(\alpha)\alpha^{-1}$.

OR

(b) Prove that $f(x) \in F(x)$ is solvable by radicals over f if and only if its splitting field E over F has solvable Galois group $G\left(\frac{E}{F}\right)$.

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