1212-15-505-002

Code No. 6379

FACULTY OF SCIENCE

M.Sc. III – Semester (CBCS) Examination, December 2016

Subject: Mathematics

Paper – IV (B)
Integral Equations

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Define Volterra integral equation and Fredholm integral equation.

2 Form an integral equation corresponding to differential equation $y''+(1+x^2)y=\cos x$, $y(0)=0, y'(0) = 0$.

3 Reduce the integral equation of the first kind to second kind $\int_0^x \sin(x-t)\,\varphi(t)dt = \sin x$.

4 Show that the solution of Abel's problem $\int_0^x \frac{\varphi(t)}{\sqrt{x-t}}dt = c$ is a cycloid.

5 Using Fredholm determinants, find the resolvent Kernal $k(x,t) = \sin x \cos t$, $a=0$, $b=2$.

6 Define characteristic numbers and eigen function of homogeneous Freedholm integral equation.

7 Show that if Kernal $K(x,t)$ is symmetyric then all its iterated Kernals are symmetric.

8 Construct the Green's function for the BVP $y''=0$, $y(0)=y(1)$, $y'(0) = y'(1)$.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Solve the integral equation $\varphi(x) = \frac{1}{1+x^2} + \int_0^x \sin(x-t)\,\varphi(t)dt$.

OR

b) Explain Picards method of successive approximation. Obtain first four approximations of $\varphi(x) = 2x^2+2 - \int_0^x x\,\varphi(t)dt$, $\varphi_0(x) = 2$.

10 a) Solve the integro-differential equation

$$\varphi''(x) + \int_0^x e^{2(x-t)}\varphi'(t)dt = e^{2x},\ \varphi(0) = 0,\ \varphi'(0) = 1.$$

OR

b) Derive and solve the Abel's problem.

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11 a) Solve the integral equation with degenerate Kernels

$$\varphi'(x) - \lambda \int_{-\pi}^{\pi} (x \cos t + t^2 \sin x + \cos x \sin t)\ \varphi(t)dt.$$

OR

b) Find the characteristic numbers and eigen function of the integral equation

$$\varphi(x) = \lambda \int_{0}^{\pi} (\cos^2 x \cos 2t + \cos 3x \cos^3 t)\ \varphi(t)dt.$$

12 a) Explain Schmidt's solution of the non-homogeneous integral equations.

OR

b) Construct the Green's function for the homogeneous BVP

$y'''(x) = 0$, $y(0) = y'(0) = y(1) = y'(1) = 0$.

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