1212-15-505-002

Code No. 6374 / CBCS

FACULTY OF SCIENCE

M. Sc. III – Semester (CBCS) Examination, December 2016

Subject : Mathematics

Paper – II : Elementary Operator Theory

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Let T be a linear operator. Define the resolvent operator, regular value, resolvent set, spectrum and spectrum value of T.

2 Prove that similar matrices have same eigen values.

3 Define a compact linear operator. If X and Y are normed spaces and If $T : X \to Y$ is a compact linear operator then show that T is bounded and hence continuous.

4 Let T be a compact linear operator on a normed space X to itself. Then show that for every $\lambda \neq 0$ the null space $N(T_\lambda)$ of $T_\lambda = T - \lambda I$ is finite dimensional.

5 Define a self adjoint operator T on a complex Hilbert space H and Hilbert adjoint Operator T* on H. Show that T is self-adjoint if and only if (Tx, x) is real.

6 Show that the residual spectrum σ_r (T) of a bounded self – adjoint linear operator $T : H \to H$ on a complex Hilbert space H is empty.

7 Show that a projection P and a Hilbert space H satisfies $0 \le P \le 1$.

8 Let P_1 and P_2 be projections on a Hilbert space H. Then show that $P = P_1 + P_2$ is a projection on H, if $Y_1 = P_1(H)$ and $Y_2 = P_2(H)$ are orthogonal.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) (i) Let $T \in B(X, X)$ where X is a Banach space. If $\|T\| < 1$ then show that $(I - T)^{-1}$ exists and $(I-T)^{-1} = \sum_{j=0}^{\infty} T^j = I + T + T^2 + \ldots\ldots$

(ii) Show that eigen vectors $x_1, x_2, \ldots. x_n$ corresponding to different eigen values $\lambda_1, \lambda_2, \ldots. \lambda_n$ of a linear operator T on a vector space X is a linearly independent set.

OR

(b) Set and prove spectral Mapping theorem for polynomials.

10 (a) Let $T : X \to X$ be a compact linear operator on a normed space X. Then show that for every $\lambda \neq 0$, the range of $T_\lambda = T - \lambda I$ is closed.

OR

(b) (i) Prove compactness of $T : \ell^2 \to \ell^2$ defined by $y = (\eta_j) = Tx$ where $\eta_j = \frac{z_j}{i}$ for j = 1, 2, ….

(ii) Let $T : X \to X$ be a compact linear operator on a normed space X and let $\lambda \neq 0$. Then prove that $Tx - \lambda x = y$ has a solution x if T(y)=0.

11 (a) (i) Show that the spectrum $\sigma(T)$ of a bounded self-adjoint linear operator $T : H \to H$ on a complex Hilbert space H is real.

(ii) Show that the spectrum $\sigma(T)$ of the above said T lies in [m, M] on the real axis where $m = \inf_{||x||=1} (Tx, x)$, $M = \sup_{||x||=1} (Tx, x)$.

OR

(b) Show that if two bounded self adjoint linear operators S and T on a Hilbert space H are positive and commute then ST is positive.

12 (a) Show that a bounded linear operator $P : H \to H$ on a Hilbert space H is a projection if and only if P is self-adjoint and idempotent.

OR

(b) Let P_1 and P_2 be projections on a Hilbert space H. Then show that $P = P_2 - P_1$ is a projection on H if and only if $Y_1 \subset Y_2$ where $Y_j = P_j(H)$.

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