

Code No. 6373

FACULTY OF SCIENCE

M.Sc. III – Semester Examination, December 2016

Subject: Mathematics

Paper – I

Complex Analysis

Time: 3 Hours

Max.Marks: 80

Note: Answer all questions from Part-A and Part-B.

Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)

[Short Answer Type]

- 1 Find the value(s) of $\tan(1 + i)$.
- 2 For what values of z is $\sum_{n=0}^{\infty} \left(\frac{z}{1+z}\right)^n$ convergent?
- 3 Find the fixed points of $T(z) = 3z$ if any.
- 4 Define a hyperbolic linear transformation and give an example.
- 5 If $f(z)$ is defined and continuous on a piecewise differentiable arc γ , then prove that

$$\left| \int_{\gamma} f dz \right| \leq \int_{\gamma} |f| |dz|.$$

- 6 Compute $\int_{|z|=2} \frac{dz}{z^2 + 1}$.

- 7 State and prove Morera's theorem.
- 8 State and prove Liouville's theorem.

PART – B (4x12 = 48 Marks)

[Essay Answer Type]

- 9 a) Prove that the sequence $\{f_n(x)\}$ converges uniformly on E if and only if to every $\epsilon > 0$ there exists on n_0 such that $|f_m(x) - f_n(x)| < \epsilon \forall m, n \geq n_0 \forall x \in E$.
OR
b) Prove that every rational function has a representation by partial fractions.
- 10 a) Define a conformal mapping. Prove that the most general transformation which leaves the origin fixed and preserves all distances is either a rotation or a rotation followed by a reflection in the real axis.
OR
b) Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.
- 11 a) State and prove Cauchy's theorem for a rectangle.
OR
b) State and prove the integral formula for higher derivatives.
- 12 a) State and prove Taylor's theorem.
OR
b) State and prove the maximum principle.
