1212-15-505-002

Code No. 8681

FACULTY OF SCIENCE

M.Sc. II-Semester Examination, May / June 2016

Subject : Mathematics

Paper - IV

Theory of Ordinary Differential Equations

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)

(Short Answer Type)

1 If f_1 and f_2 are linearly independent functions on an interval I then prove that $f_1 + f_2$ and $f_1 - f_2$ are also linearly independent on I.

2 If $\phi_1, \phi_2, \ldots, \phi_n$ are n linearly independent solution and ϕ is any solution of the n^{th} order equation $L(x(t)) - x^{(n)}(t) + b_1(t)\, x^{(n-1)}(t) + \ldots\ldots + b_n(t)\, x(t) = 0$, $t \in I$, then prove that there exists n constants $c_1, c_2, \ldots, c_n$ such that $\phi = c_1\phi_1 + \ldots + c_n\phi_n$, $t \in I$.

3 Write general n^{th} order equation $x^{(n)} = g(t, x, x^1, \ldots, x(h-1)$, $t \in I$., into a system of first order n equations.

4 If ϕ is a fundamental matrix for the system $x^1 = A(t)\, x$ and if C in a constant non singular matrix then prove that ϕC is also a fundamental matrix for above system.

5 Show that the function $f(t, x) = x^2 + \cos^2 t$ satisfies Lipschitz condition in the region R : $\{(t, x) : 0 \le t \le a\ , |x| \le b\}$.

6 Define contraction mapping and also prove that every contraction mapping has unique fixed point.

7 Define equicontinuous and uniformly bounded of a family of function $\{f_\alpha(t)\}$ defined on I and also state Ascoli's Lemma.

8 If $v, w \in C'\{[t_0, t_0 + h], R\}$ are lower and upper solutions of IVP $x^1 = f(t,x)$, $x(t_0) = x_0$ and if f satisfies the inequality $f(t,x) - f(t,y) \le L(x-y)$ for $x \ge y$ then prove that $v(t_0) \le w(t_0)$ implies $v(t) \le w(t)$ on $I = [t_0, t_0 + h]$.

PART – B (4 x 12 = 48 Marks)

(Essay Answer Type)

9 a) State and prove Able's formula for nth order linear homogene differential equation $L(x)(t) = x^{(n)}(t) + b_1(t)x^{(n-1)}(t) + \ldots + b_n(t)x(t) = 0$, $t \in I$, where $b_1, b_2, \ldots, b_n$ are continuous functions defind on an interval I.

OR

b) Solve $x'' - 7x' = (3 - 36t)e^{4t}$ using the method of undetermined coefficients.

10 a) If A(t) is an n x n matrix continuous on I and if matrix Φ satisfies $X^1 = A(t)X$, $t \in I$ then prove that $\det\Phi$ satisfies the first order equation $(\det \Phi)' = (\text{tr}A)(\det \Phi)$.

OR

b) Determine the fundamental matrix for the system $x' = Ax$, where

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$

11 a) State and prove Picard's theorem for existence and uniqueness of solution of IVP $x' = f(t, x), x(t_0) = x_0$.

OR

b) Prove that IVP $x' = f(t, x)$, $x(t_0) = x_0$ has unique solution on $[t_0, t_0+h]$ using contraction principle if f(t, x) is continuous on the strip $t_0 \le t \le t_0 + h$, $|x| < \infty$ and f satisfies Lipschitz condition wilt Lipschitz constant $K > 0$.

12 a) If f in IVP $x' = f(t, x), x(t_0) = x_0$, is non increasing in x, then prove that the iterates Vn(t) given by $V'_{n+1} = f(t, v_n)$, $v_{n+1}(t_0) = x_0$, and the unique solution x(t) of above IVP satisfy the inequality $v_0(t) \le v_2(t) \le \ldots\ldots\ldots \le x(t) \le \ldots\ldots \le v_3(t) \le v_1(t), t \in I$ provided $v_2(t) \ge v_0(t)$ and also the sequence $\{v_{2n}(t)\}$, $\{v_{2n+1}(t)\}$ converge uniform to $\rho(t)$, r(t) and $\rho(t) \le x(t) \le r(t)$, $t \in I$.

OR

b) Prove that IVP $x^1 = f(t, x)$, $x(t_0) = x_0$ has unique solution on the strip $S = \{(t, x)\ ;\ t_0 \le t \le t_0 + h, |x| < \infty\}$ if f is continuous and bounded on S using Ascoli's lemma.

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