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Code No. 8679

FACULTY OF SCIENCE

M.Sc. II-Semester Examination, May / June 2016

Subject : Mathematics / Mathematics with Computer Science

Paper - III : Functional Analysis

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)

(Short Answer Type)

1 Prove that the Euclidean space R^n is a Banach space.

2 Let in be a non-empty subset of a metric space X. Prove that M is closed if and only if $x_n \in M,\ x_n \to x \Rightarrow x \in M$.

3 Prove that inner product is jointly continuous.

4 Prove that the space c[a, b] is not a Hilbert space.

5 Let X and Y be inner product spaces and A : X → Y a bounded linear operator. Then show that A = 0_{OP} (zaro operator) if and only if < Ax, y >=0 for all x ∈ X and all y ∈ Y.

6 Let S and T be bounded linear operators on a Hilbert space H and α any scalar. Then prove that i) $(S+T)^* = S^* + T^*$ ii) $(\alpha T)^* = \bar{\alpha} T^*$

7 Let X and Y be normed spaces and S, T ∈ B(X, Y). Then prove that
i) $(S+T)^x = S^x + T^x$ ii) $(\alpha T)^x = \alpha T^x$ for all scalars α.

8 Let X and Y be normed spaces and T : D ⊂ X → Y be a linear operator. Prove that T is closed if and only if the following condition holds $x_n \in D$, $x_n \to x \in X$ and $Tx_n \to y \in Y \Rightarrow x \in D$ and y = Tx.

PART – B (4 x 12 = 48 Marks)

(Essay Answer Type)

9 a) Let $\{x_1, x_2, \ldots, x_n\}$ be a linearly independent set of vectors in a normed space X (of any dimension). Then prove that there is a number c > 0 such that for every choice of scalars α1, α2, ….., αn, we have
$\|\alpha_1 x_1 + \alpha_2 x_2 + \ldots\ldots + \alpha_n x_n\| \geq C(|\alpha_1| + |\alpha_2| + \ldots\ldots + |\alpha_n|)$.

OR

b) Prove that the vector space B(X, Y) of all bounded linear operators from a normed space X into a Banach space Y is a Banach space.

10 a) i) Let Y be a complete subspace of an inner product space X and x ∈ X fixed. Then prove that z = x – y is orthogonal to Y.
ii) If Y is a closed subspace of a Hilbert space H, then prove that Y = $Y^{\perp\perp}$.

OR

b) State and prove Schwerz inequality and triangle inequality.

11 a) Prove that two Hilbert spaces H and $\tilde{H}$, both real or both complex, are isomorphic if and only if they have the same Hilbert dimension.

OR

b) State and prove Riesz representation theorem for sesqui linear form.

12 a) State and prove uniform boundedness theorem.

OR

b) State and prove closed graph theorem.

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