1212-15-505-002

Code No. 8678

FACULTY OF SCIENCE

M.Sc. II-Semester Examination, May / June 2016

Subject : Mathematics / Applied Mathematics

Paper - II

Advanced Real Analysis

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)

(Short Answer Type)

1 Prove that if $m^*(A)=0$ then $m^*(A \cup B) = m^*(B)$.

2 Prove that if f is a measurable function and f = g a.e. then g is measurable.

3 Show that if

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irratioal} \\ 1 & \text{if } x \text{ is rational} \end{cases}$$

then $R\int_a^{-b} f(x)dx = b - a$ and $R\int_{-a}^{b} f(x)dx = 0$.

4 Show that if f is integrable over E, then so is |f| and $\left|\int_E f\right| \le \int_E |f|$.

5 Show that $D^+[-f(x)] = -D_+f(x)$.

6 Prove that every convergent sequence is a Cauchy sequence.

7 Consider the mapping $\bar{f} = (f_1, f_2)$ of IR^5 into IR^2 given by

$f_1(x_1, x_2, y_1, y_2, y_3) = 2e^{x_1} + x^2 y_1 - 4y_2 + 3.$

$f_2(x_1, x_2, y_1, y_2, y_3) = x_2 \cos x_1 - 6x_1 + 2y_1 - y_3.$

If $\bar{a} = (0,1)$ and $\bar{b} = (3, 2, 7)$ then find the values of $\bar{f}(\bar{a}, \bar{b})$.

8 Put f(0,0) = 0, and

$$f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2} \text{ if } (x, y) \neq (0, 0)$$

Show that $D_{12}f$ and $D_{21}f$ exist at every point of IR^2, but that $D_{12}f \neq D_{21}f$ at (0, 0).

PART – B (4 x 12 = 48 Marks)

(Essay Answer Type)

9 a) i) Prove that the interval (a, ∞) is measurable.

ii) Let {En} be an infinite decreasing sequence of measurable sets, that is, a sequence with $E_{n+1} \subset E_n$ for each n. Let $m(E_1)$ be finite. Then prove that

$$m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n).$$

OR

b) i) Let f and g be two measurable real-valued functions defiend on the same domain. Then prove that f + g is also measurable.

ii) State and prove Littlewood's third principle.

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10 a) i) Let f be a bounded function defined on [a, b]. If f is Riemann integrable on [a, b] then prove that it is measurable and

$$R\int_a^b f(x)dx = \int_a^b f(x)dx.$$

ii) Define integral of a nonnegative function. Prove that if f and g are nonnegative measurable functions then

$$\int_E (f+g) = \int_E f + \int_E g$$

OR

b) i) Let f be a nonnegative function which is integrable over a set E. the prove that given $\in > 0$ there is a $\delta > 0$ such that for every set $A \subset E$ with $m(A) < \delta$ we have

$$\int_A f < \in$$

ii) Let $\{f_n\}$ be a sequence of measurable functions that converges in measure to f. Then prove that there is a subsequence $\{f_{n_k}\}$ that converges to f almost everywhere.

11 a) Let f be an increasing real-valued function on the interval [a, b]. Then prove that f is differentiable almost everywhere. Also prove that the derivative f' is measurable and $\int_a^b f'(x)dx \le f(b) - f(a)$.

OR

b) Prove that L^p spaces are complete.

12 a) State and prove the rank theorem.

OR

b) i) Suppose f is defined in an open set set $E \subset IR^2$, and D_1f and $D_{21}f$ exist at every point of E. Suppose $Q \subset E$ is a closed rectangle with sides parallel to the coordinate axes, having (a, b) and (a+h, b+k) as opposite vertices ($h \neq 0$, $k \neq 0$). Put $\Delta(f,Q) = f(a+h,b+k) - f(a+h,b) - f(a,b+k) + f(a,b)$. Then prove that there is a point (x, y) in the interior of Q such that

$$\Delta(f,Q) = hk(D_{21}f)(x,y).$$

ii) Prove that $D_{21}f = D_{12}f$ if $f \in \mathscr{C}''(E)$.

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