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Code No. 8677

FACULTY OF SCIENCE

M.Sc. II-Semester Examination, May / June 2016

Subject : Mathematics / Applied Mathematics

Paper - I : Advanced Algebra

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 If p(x) is an irreducible polynomial in F(x) then prove that there exists an extension E of F in which p(x) has a root.

2 Show that $x^2 - x - 1 \in \frac{z}{(3)}$ is irreducible over $\frac{z}{(3)}$.

3 Prove that $x^p - 1 \in Q(x)$ has splitting field $Q(\alpha)$ where $\alpha \neq 1$ and $\alpha^p = 1$. Also show that $[Q(\alpha):Q] = p-1$ where p is a prime.

4 If the multiplicative group F* of nonzero elements of a filed F is cyclic then prove that F is finite.

5 If F is a field of characteristic $\neq 2$ and $x^2 - a \in F(x)$ is an irreducible polynomial over F then prove that its Galois group is of order 2.

6 Prove that the Galois group of $x^4 + 1 \in Q(x)$ is the Klein four group.

7 Prove that the Galois group of $x^4 + x^2 + 1$ and $x^6 - 1$ is same and is of order 2.

8 Show that the polynomial $x^5 - 9x + 3$ is not solvable by radicals over Q.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 a) For any field K prove that the following are equivalent.

i) K is algebraically closed

ii) Every irreducible polynomial in k(x) is of degree one

iii) Every polynomial is k(x) of positive degree factors completely in k(x) into linear factors.

iv) Every polynomial in k(x) of positive degree has atleast one root in K.

OR

b) If E is an algebraic extension of a field F and $\sigma : F \rightarrow L$ is an embedding of F into an algebraically closed filed L then prove that σ can be extended to an embedding $\eta : E \rightarrow L$.

10 a) If E is a finite seperable extension of a field F then prove that E is a simple extension of F.

OR

b) If E is a finite extension of a field F prove that the following are equivalent.

i) $E = F(\alpha)$ for some $\alpha \in E$

ii) There are only a finite number of intermediate fields between F and E.

11 a) State and prove fundamental theorem of Galois theory.

OR

b) State and prove fundamental theorem of algebra.

12 a) i) If F is field and U is a finite subgroup of the multiplicative group F* = F – {0} then prone that U is cyclic.

ii) Show that $\phi_8(x)$ and x^8-1 have the same Galois group $\left(\frac{Z}{(8)}\right)^* = \{1,3,5,7\}$.

OR

b) Prove that f(x) $\in$ F(x) is solvable by radicals over F if and only if its splitting field E over F has solvable Galois group $G\left(\frac{E}{F}\right)$.

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