1212-16-505-001

Code No. 6070/CBCS

FACULTY OF SCIENCE

M. Sc. I – Semester (CBCS) Examination, December 2016

Subject : Mathematics

Paper – IV : Elementary Number Theory

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Use recursion to find the gcd of 18, 30, 60, 75, 132.

2 If f_n denotes the nth Fermat number, show that $641/ f_5$.

3 Find the digital roots of square numbers.

4 Solve the linear system

$x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 4$, $x \equiv 3 \pmod 5$

using Chinese Remainder theorem.

5 Find the remainder when 24^{1947} is divided by 17.

6 Computer $\phi(8)$, $\phi(81)$, $\phi(15625)$ where ϕ is Euler's Phi function.

7 Verify that 2 is a primitive root modulo 9.

8 Solve the quadratic congruence $3x^2 - 4x + 7 \equiv 0 \pmod 3$.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) Solve the linear Diophantine equation $1076x + 2076y = 3076$ by Euler's method.

OR

(b) Show that LDE $ax + by = c$ is solvable if and only if d/c where $d = (a, b)$. If x_o, y_o is particular solution of LDE then all its solutions are given by

$x = x_o + \frac{b}{d}t$ and $y = y_o - \frac{a}{d}t$ where t is an arbitrary integer.

10 (a) Using Pollard Rho method, factor the integer 3893.

OR

(b) (i) Using the method of elimination, solve the linear system

$2x + 3y \equiv 4 \pmod{13}$
$3x + 4y \equiv 5 \pmod{13}$

(ii) Prove that digital root of the product of twin primes, other than 3 and 5 is 8.

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11 (a) (i) State and prove Euler's theorem.

(ii) Deduce Fermat's Little theorem from Euler's theorem.

OR

(b) (i) If n is a positive integer, show that $\sum_{d/n} \phi(d) = n$.

(ii) Verify that M_{11} is a composite number and determine if M_{19} is a prime where Mp denotes Mersenne prime.

12 (a) (i) Show that α=3 is a primitive root modulo 5 and 5^2. Also prove that α=5 is a primitive root modulo 7 and α+p=5+7=12 is a primitive root modulo 7^2.

(ii) Solve the congruence $3x^2 - 4x + 7 \equiv 0 \pmod{13}$.

OR

(b) (i) State and prove Law of Quadratic reciprocity.

(ii) Evaluate (2 | 13) using Gauss Lemma.

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