1212-16-505-001

Code No. 6066/CBCS

FACULTY OF SCIENCE

M. Sc. I – Semester (CBCS) Examination, December 2016

Subject : Mathematics/Applied Mathematics / Maths with Computer Science

Paper – I : Algebra

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1. Prove that every finite group has a composition series.
2. Show that every homomorphic image of a solvable group is solvable.
3. Prove that a finite group G is a P-group iff its order is power of P.
4. Show that a group of order 1986 is not simple.
5. Suppose $f : F \rightarrow R$ is a non zero homomorphism of a field F into a ring R. Then prove that f is one-one.
6. Show that if A and B are nilpotent ideals, their sum A + B is nil potent.
7. Prove that an irreducible element in a commutative principle ideal domain is always prime.
8. If $f(x), g(x) \in R(x)$ then prove that $C(fg) = C(f)C(g)$, where $C(f)$ denotes the content of $f(x)$.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9. (a) Derive class equation of a finite group.

OR

(b) Prove that a group G is solvable if and only if G has a normal series with abelian factors.

10. (a) (i) State and prove Cauchys theorem for finite abelian groups.
(ii) Prove that there is a1 – 1 correspondence between the family F of non-isomorphic abelian groups of order P^e. P prime, and the set P(e) of a partitions of e.

OR

(b) (i) If the order of a finite group G is divisible by a prime power P^α, then prove that G has subgroup of order P^α.
(ii) Prove that a a sylow P-subgroup of a finite group G is unique if and only if it is normal.

11. (a) State and prove correspondence theorem for rings.

OR

(b) If R is a non-zero ring with unity and I is an ideal in R such that $I \neq R$ then prove that there exists a maximal ideal M of R such that $I \subseteq M$.

12. (a) Prove that every PID is a UFD but a UFD need not be a PID.

OR

(b) If R is a unique factorization domain, then prove that the polynomial ring R[x] over R is also a unique factorization domain.

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