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Code No. 8044

FACULTY OF SCIENCE

M. Sc. I – Semester Examination, December 2015

Subject : Mathematics

Paper – III : Topology

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 If A and B are subsets of a topological space X then prove the following:

(i) $\overline{\phi} = \phi$ (ii) $A \subseteq \overline{A}$ (iii) $\overline{\overline{A}} = \overline{A}$ (iv) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

2 Define separable space with an example.

3 Show that a topolotical space is compact if and only if every class of closed sets with the finite intersection property has non-empty intersection.

4 Show that a closed subspace of a complete metric space is compact if and only if it is totally bounded.

5 Show that every compact Hausdorff space is normal.

6 Show that a topological space is a T_1-Space if and only if each point is a closed set.

7 Show that product of any non-empty class of Hausdorff spaces is a Hausdorff space.

8 Show that a topological space is disconnected if and only if there exists a continuous mapping of X onto the discrete two point space {0, 1}.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) (i) Define open subbase.

(ii) Let X be any non-empty set and let S be an arbitrary class of subsets of X. Then show that S can serve as an open subbase for a topology on X, in the sense that the class of all unions of finite inter sections of sets in S is a topology.

OR

(b) (i) Let X and Y be topological spaces and f: X $\rightarrow$ Y be a mapping and let there be given an open base in X and an open subbase with its generated open base in Y. The prove that f is continuous $\Leftrightarrow$ the inverse image of each basic open set is open.

$\Leftrightarrow$ the inverse image of each subbasic open set is open

10 (a) (i) Show that in a sequentially compact metric space, every open cover has a Lebesgue number.

(ii) Show that a metric space is compact if and only if it is complete and totally bounded.

OR

(b) (i) State and prove Ascoli's theorem.

(ii) Show that every sequentially compact metric space is totally bounded

11 (a) State and prove Tietze extension theorem.

OR

(b) State and prove Urysohn imbedding theorem.

12 (a) (i) Show that a subspace of the real line R is connected if and only if it is an interval.

(ii) Show that range of a continuous real function defined on a connected space is an interval.

OR

(b) If X is an arbitrary topological space then prove that following:

(i) Each point in X is contained in exactly one component of X.

(ii) Each connected subspace of X is contained in a component of X.

(iii) A connected subspace of X which is both open and closed is a component of X.

(iv) Each component of X is closed.

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