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Code No: 7114/E

FACULTY OF SCIENCE

B.Sc. IV – Semester (CBSC) Examination, May / June 2018

Subject: Mathematics

Paper: IV Algebra

Time: 3 Hours **Max. Marks: 80**

SECTION – A (5 x 4 = 20 Marks)

(Short Answer Type)

Note: Answer any Five of the following questions

1. Let G be any group and $(ab)^2 = a^2 b^2$ for all a, b $\in$ G then show that G is an abelian group. 69

2. If $\alpha, \beta \in S_5$ and α = (1 2 3 4 5), β = (1 4 5 3 2) then evaluate $\alpha\beta$, $\alpha\beta^{-1}, \alpha^2\beta$.

3. If H and K are subgroups of a group G with $|H| = 24, |K| = 20$ then show that $H \cap K$ is an abelian group.

4. Determine all group homomorphisms from $\mathbf{Z}_{12}$ to $\mathbf{Z}_{30}$.

5. Define zero divisor in a ring R. Find all zero divisors in the ring ($\mathbf{Z}_{12}$, $+_{12}$, $\cdot_{12}$)

6. If I_1 and I_2 are any two ideals in a ring R, then show that $I_1 + I_2 = \{x + y / x \in I_1, y \in I_2\}$ is always an ideal of R.

7. Show that $f(x) = x^2 + 3x + 2$ has four zeros in $\mathbf{Z}_6$.

8. Let R, S be any two rings and $\phi : R \to S$ is a homo morphism. If R is communicative then show that ϕ(R) is commutative.

SECTION-B (4x15=60 Marks)

(Essay Answer Type)

9. (a) (i) Let G be a group and H is non empty subset of G. Then show that H is group of G if and only if $ab^{-1} \in$ H for all a,b $\in$ H.
   (ii) In the symmetric group S_3 find the elements which satisfy $x^3 = e$ where e is the identity permutation of S_3.

OR

   (b) (i) Show that every subgroup cyclic group is cyclic.
   (ii) Find all generators of the group ($\mathbf{Z}_8$, $+_8$).

Contd…2….

10. (a) (i) State and prove Lagrange's theorem.
(ii) Show that every group of prime order is cyclic.

OR

(b) (i) Define center z(G) of a group G. Show that Z(G) is always a normal subgroup of G.
(ii) State and prove the first isomorphism theorem on groups.

11. (a) (i) Show that every finite integral domain is a field.
(ii) If a is an idempotent element in a ring R then show that 1-a is also idempotent.

OR

(b) (i) Define Maximal ideal in a ring R.
(ii) Let R be a cumulative ring with unity and A be an ideal of R then show That quotient $\frac{R}{A}$ is a field if and only if A is a maximal ideal.

12. (a) Let D be an integral domain. Then show that there exists a field F that contains a subring isomorphic to D.

OR

(b) Let F be a field and $f(x), g(x) \in F[x]$ with $g(x) \neq 0$. Then show that there exists unique polynomials q(x) and r(x) in F[x] such that $f(x) = q(x)g(x) + r(x)$ with either $r(x) = 0$ or $\deg r(x) < \deg g(x)$.

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