Code No. 7068 / E

FACULTY OF SCIENCE

B.Sc. III-Semester (CBCS) Examination, December 2017

Subject: Mathematics

Paper – III
Real Analysis

Time: 3 Hours **Max.Marks: 80**

PART – A (5x4 = 20 Marks)
[Short Answer Type]

Note: Answer any FIVE of the following questions.

1 Prove that $\lim_{n\to\infty}\left(n^{\frac{1}{n}}\right) = 1$.

2 Prove that every convergent sequence is a Cauchy sequence.

3 Let $\{s_n\}$ be a sequence converging to s. Then prove that $\lim_{n\to\infty}\sigma_n = s$, where $\sigma_n = \frac{1}{n}(s_1 + s_2 + \ldots + s_n)$.

4 If a series $\sum a_n$ converges, then show that $\lim_{n\to\infty} a_n = 0$.

5 Find the radius of convergence of $\sum_{n=1}^{\infty}\left(\frac{3^n}{n.4^n}\right)x^n$.

6 Let $\{f_n\}$ be a sequence of continuous functions on [a,b] and suppose that $f_n \to f$ uniformly on [a,b]. Then prove that $\lim_{n\to\infty}\int_a^b f_n(x)\,dx = \int_a^b f(x)\,dx$.

7 If f is a bounded function on [a,b], and if P and Q are partitions of [a,b], then prove that $L(f,P) \leq U(f,Q)$.

8 Let f(x) = x for rational x and f(x) = 0 for irrational x. Calculate the upper and lower Darboux integrals for f on the interval [a,b].

PART – B (4x15 = 60 Marks)
[Essay Answer Type]

Note: Answer ALL the questions.

9 a) i) If $\{s_n\}$ converges to s and $\{t_n\}$ converges to t then prove that s_n+t_n converges to s+t.

ii) Prove that a bounded monotone sequence converges.

OR

b) i) Prove that every Cauchy sequence is bounded.

ii) Prove that every Cauchy sequence of real numbers is convergent

10 a) Let $\{s_n\}$ be a sequence, $t \in \mathbb{R}$. Then prove that there is a subsequence of $\{s_n\}$ converging to t if and only if the set $\{n \in \mathbb{N} : |s_n - t| < \epsilon\}$ is infinite for each $\epsilon > 0$.

OR

b) i) If the sequence $\{s_n\}$ converges, then prove that every subsequence converges to the same limit.

ii) Prove that every sequence has monotone subsequence.

11 a) i) Find the radius of convergence of the series $\sum_{n=1}^{\infty} x^{n!}$.

ii) Prove that the uniform limit of continuous functions is continuous.

OR

b) i) State and prove Weierstrass M-test.

ii) Show that if the series $\sum g_n$ converges uniformly on a set s, then

$$\lim_{n \to \infty} \sup \{ |g_n(x)| : x \in s\} = 0.$$

12 a) Define Riemann integral $\int_a^b f(x)\,dx$. If f is a bounded function on [a,b], then prove that $L(f) \leq U(f)$.

OR

b) Prove that a bounded function f on [a,b] is integrable if and only if for each $\epsilon > 0$ there exists a partition P of [a,b] such that $U(f,P) - L(f,P) < \epsilon$.

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