Code No. 228 / E

FACULTIES OF ARTS AND SCIENCE

B.A. / B.Sc. I-Semester (CBCS) Examination, December 2016

Subject : Mathematics

Paper – I : Differential Calculus

Time : 3 Hours Max. Marks: 80

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)
Note : Answer any FIVE of the following questions.

1 Expand f(x) = log (1 + sinx) by using Maclaurin's theorem.

2 Find the c value of Rolle's mean value theorem for the function
$f(x) = \log\left(\frac{x^2+ab}{x(a+b)}\right)$ on [a, b].

3 Evaluate $\lim_{x\to 0}(\cot x)^{\sin x}$.

4 Find the radius of curvature of the curve
$x = a\cos^3 t$, $y = b\sin^3 t$ at $t = \pi/4$.

5 If $z = \log(u^2 + v)$, $u = e^{x+y^2}$, $v = x + y^2$ then evaluate $2y\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}$

6 If $u = Tan^{-1}\left(\frac{y^2}{x}\right)$ then evaluate
$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

7 Find the asymptotes of the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$, which are parallel to coordinate axes.

8 Find the envelope of the curve.
$my + m^2 x - 10 = 0$ where m is a parameter.

PART – B (4 x 15 = 60 Marks)
(Essay Answer Type)
Note: Answer ALL the questions.

9 (a) (i) y = a cos (logx) + bsin(logx) then show that
$x^2y_{n+2} + (2n + 1)\, xy_{n+1} + (n^2+1)y_n = 0$

(ii) Find the coefficient of x^5 in Maclaurin's series expansion of $f(x)=e^x \cos x$.

OR

(b) (i) State and prove Cauchy's mean value theorem. Hence find 'c' value of Cauchy's mean value theorem for the function $f(x)=e^x$, $g(x) = e^{-x}$ on [a, b].

10 (a) (i) Find the curvature, the radius of curvature and the centre of the circle of curvature and the circle of curvature for the curve $x^2 = 4ay$ at P(2a, a).

(ii) Find the evolute of the parabola $y^2 = 4ax$.

OR

(b) (i) Evaluate $\lim_{x\to 0} \frac{(1+x)^{1/x} - e + \frac{ex}{2}}{x^2}$.

(ii) Find the value of a and b so that

$$\lim_{x\to 0} \frac{x(1+a\cos x) - b\sin x}{x^2} = 1$$

11 (a) (i) If $f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 \quad , & (x,y) = (0,0) \end{cases}$

then show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

(ii) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

OR

(b) (i) State and prove Euler's theorem for a homogeneous function.

(ii) Explain $f(x,y) = x^2y + 3y - 2$ in terms $x+1, y-2$ as a Taylor series.

12 (a) (i) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$ where a > 0.

(ii) Discuss the maximum and minimum values of

$$f(x,y) = xy + \frac{9}{x} + \frac{3}{y}$$

OR

(b) Find the asymptoptes of the curve.

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$$

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