1212-17-505-011

Code No. 7404 / CORE

FACULTY OF SCIENCE

M.Sc. III – Semester Examination, Dec. 2018 / Jan. 2019

Subject: Mathematics

Paper – I
Complex Analysis

Time: 3 Hours | **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B. Each question carries 4 marks in part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Find all roots of cos z = 2.

2 Find $|\cosh z|^2$

3 Show that if C is a positively oriented simple closed contour, then the area of the region enclosed by C is $\frac{1}{2i}\int_C \bar{z}\, dz$.

4 State and prove the principle of deformation of paths.

5 Show that $\sum_{n=1}^{\infty} r^n \cos n\theta = \frac{r\cos\theta - r^2}{1-2r\cos\theta + r^2}$, where $0 < r < 1$.

6 Find the Laurent series of $f(z) = \frac{1}{z^2(1-z)}$, $1 < |z| < \infty$.

7 Compute the linear fractional transformation that maps 2, i, -2 onto 1, i, -1.

8 Find the fixed points of $w = \frac{z-1}{z+1}$.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Find the harmonic conjugates of
   i) $u(x,y) = \sinh x \sin y$
   ii) $u(x,y) = 2x - x^3 + 3xy^2$.

OR

b) i) State and prove the reflection principle.
   ii) Prove that, a function that is analytic in a domain D is uniquely determined by its values in a domain or along a line segment contained in D.

10 a) i) Prove Cauchy's inequality.
   ii) Prove Liouville's theorem.

OR

b) i) Prove maximum modulus principle.
   ii) Find the absolute maximum of f(z) = sin z over the region $[0, \pi] \times [0, 1]$.

11 a) State and prove Laurent's theorem.

OR

b) Prove that $\operatorname{Res}_{z=\infty} f(z) = -\operatorname{Res}_{z=0}\left(\frac{1}{z^2} f\left(\frac{1}{z}\right)\right)$ and hence evaluate $\int_{|z|=2} \frac{5z-2}{z(z-1)} dz$, |z|=2 is positively oriented.

12 a) Evaluate $\int_0^\infty \frac{x^2}{1+x^6} dx$.

OR

b) Evaluate P.V. $\int_{-\infty}^{\infty} \frac{(x+1)\cos x}{x^2+4x+5}$ dx.

****

1212-17-505-011)

Code No. 7406 / CORE

FACULTY OF SCIENCE

M.Sc. III – Semester Examination, Dec. 2018 / Jan. 2019

Subject: Mathematics

Paper – II
Functional Analysis

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Let E be a complete normed linear space over IR or ¢. If $\{x_n\}, \{y_n\} \subset E_n, \alpha_n \in$ IR or ¢ and $x_n \to x$, $y_n \to y$ respectively as $n \to \infty$ and $\alpha_n \to \alpha$ as $n \to \infty$ then prove that
   i) $x_n + y_n \to x+y$
   ii) $\alpha_n x_n \to \alpha x$ as $n \to \infty$

2 Prove that on a finite dimensional normed linear space any norm $||.||$ is equivalent to any norm $||.||'$.

3 State that prove parallelogram law.

4 Prove that the space ℓ_p with $p \neq 2$ is not a Hilbert space.

5 Prove that every linear functional is homogeneous.

6 Let E_x be a normed linear space over IR or ¢. If A, B ϵ $(E_x \to E_x)$ then prove that $AB \epsilon (E_x \to E_x)$ and $||AB|| \leq ||A|| \, ||B||$.

7 Define projection operator. Also prove that if P is projection then I-P is projection.

8 Define:
   i) Normal operator
   ii) Unitary operator.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Let $\{e, e_2, \ldots, e_n\}$ be a linearly independent set of vectors in a normed linear space E (of any dimension) then there is a number C > 0 such that for any choice of scalars $\alpha_1, \alpha_2, .., ., \alpha_n$, we have
$||\alpha_1 e_1 + \alpha_2 e_2 + \ldots + \alpha_n e_n|| \geq c\,(|\alpha_1| + |\alpha_2| + \ldots + |\alpha_n|)$

OR

b) Let Y and Z be subspaces of a normed linear space X and suppose that Y is closed and is a proper subset of Z then prove that for every $\theta \in (0,1)$ there is a $z \epsilon Z$ such that $||z|| = 1$, $||z-y|| \geq \theta$ for all $y \, \varepsilon \, Y$.