1212-18-505-021

Code No. 8192 / Core

FACULTY OF SCIENCE

M.Sc. II – Semester Examination, May / June 2019

Subject: Mathematics / Applied Maths

Paper – I
Galois Theory

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Let $f(x) \in F[x]$ be a polynomial of degree >1. If $f(\alpha) = 0$ for some of $\alpha \in F$ then f(x) is reducible over F.

2 If E is a finite extension of F then show that E is an algebraic extension of F.

3 Let $F = \frac{z}{(2)}$ then prove that the splitting field of $x^3+x^2+1 \in E[x]$ is a finite field with eight elements.

4 Let F be a finite field then prove that the number of elements of F is p^n for some positive integer n and p is prime number.

5 Prove that $G = G\left(\frac{\mathbb{C}}{R}\right)$ is a group of R-automorphisms of ¢ and $|G| = 2$.

6 Prove that the group $G\left(\frac{Q(\alpha)}{Q}\right)$, where $\alpha^5 = 1$ and $\alpha \neq 1$ is isomorphic to the cyclic group of order 4.

7 Prove that $\phi_8(x)$ and x^8-1 have the same Galois group, namely $\left(\frac{Z}{(8)}\right)^* = \{1, 3, 5, 7\}$, the Klein four group.

8 Show that $x^5 - 9x + 3$ is not solvable by radicals over Q.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Let $F \subseteq E \subseteq K$ be fields. If $[K:E] < \infty$ and $[E:F] < \infty$ then show that

i) $[K:F] < \infty$

ii) $[K:F] = [K:E]\,[E:F]$

OR

b) For any field K the following are equivalent

i) K is algebraically closed

ii) Every irreducible polynomial in K[x] is of degree 1

iii) Every polynomial in K[x] of positive degree factors completely in K[x] into linear factors

iv) Every polynomial in K[x] of positive degree has atleast one root in K.

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10 a) Prove that the degree of the extensions of the splitting field of $x^3 - 2 \in Q[x]$ is 6.

OR

b) If E is a finite separable extension of a field F then prove that E is a simple extension of F.

11 a) Let H be a finite subgroup of the group of automorphisms of a field E then $[E:E_H] = |H|$.

OR

b) State and prove fundamental theorem of Galois Theory.

12 a) Let F contains a primitive n^{th} root w unity then the following are equivalent.

I) E is a finite cyclic extension of degree n over F

ii) E is the splitting field of an irreducible polynomial $x^n - b \in F[x]$

Further more $E = F(\alpha)$ where α is a root of $x^n - b$.

OR

b) Prove that the following are equivalent statements:

i) $a \in R$ is constructible form Q

ii) (a, 0) is a constructible point from QxQ

iii) (a, a) is a constructible point form QxQ

iv) (0, a) is a constructible point from QxQ.

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