12121950SQ24

Code No. 1268/CORE

FACULTY OF SCIENCE

M. Sc. II – Semester Examination, December 2020

Subject : Mathematics / Applied Maths

Paper – I : Galois Theory

Time : 2 Hours **Max. Marks: 80**

Note: Answer any five questions. **(5x7=35 Marks)**

1 Let f(x) $\in$ F[x] be a polynomial of degree 2 or 3 then show that f(x) is reducible if and only if f(x) has a root in F.

2 Let f(x) $\in$ F[x] be a non-constant polynomial then there exists an extension E of F in which f(x) has a root.

3 Let f(x) $\in$ F[x] be a polynomial of degree ≥ 1 with α as a root then α is a multiple root if and only if f′(α) = 0.

4 Prove that the multiplicative group of non zero elements of a finite field is cyclic.

5 Prove that $G = G\left(\frac{Q(3\sqrt{2})}{Q}\right)$ is a group of Q-auto morphisms of Q$(3\sqrt{2})$ and |G| = 1.

6 Define fixed field of H. If $H = G\left(\frac{C}{R}\right)$ then prove that C_H=R.

7 Let F be a field and let U be a finite subgroup of the multiplicative group F* = F – {0} then prove that U is cyclic.

8 If a and b are constructible numbers then a ± b are also constructible.

PART – B

Note: Answer any three questions. **(3x15=45 Marks)**

9 (i) Let $f(x) = a_o + a_{1x} + ... + a_{n-1}x^{n-1} + x^n \in z[x]$ be a monic polynomial. If f(x) has a root $\alpha \in$Q then $a \in$ Z and a/a_0.

(ii) State and prove Eisenstein criterion.

10 Let E be an extension field of F and let u$\in$E be algebraic over F. Let p(x) $\in$ F[x] be a polynomial of the least degree such that p(u) = 0 then show that

(i) p(x) is irreducible over F

(ii) If g(x) $\in$ F[x] is such that g(u) = 0 the p(x)/ g(x)

(iii) there is exactly one monic polynomial p(x) $\in$ F[x] of least degree such that p(u) = 0

11 If f(x) $\in$ F[x] is irreducible over F then prove that all roots of f(x) have the same multiplicity.

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12 Let E be a finite extension of a field F then show that the following are equivalent
(i) E = F (α) for some $\alpha \in$ E
(ii)There are only a finite number of intermediate fields between F and E.

13 Let E be a finite separable extension of a field E then show that the following are equivalent
(i) E is a normal extension of F
(ii)F is a the fixed field of $G\left(\frac{E}{F}\right)$
(iii) [E : F] = | $G\left(\frac{E}{F}\right)$ |

14 Let F be a field of characteristic $\neq$ 2 or 3 Let f(x) = x^3 + bx + c be a separable polynomial over F. If f(x) is irreducible over F then the Galois group of f(x) is of order 3 or 6. Also then Galois group of f(x) is S_3 if and only if $\Delta = - 4b^3 - 27C^2$ is not a square in F, that is, there does not exist only element $\alpha \in$ F such that $\alpha^2 = \Delta$.

15 Let f(x) $\in$ F[x] is solvable by radical over F if and only if its splitting field E over F has solvable Galois group $G\left(\frac{E}{F}\right)$

16 (i) Prove that it is impossible to construct a square equal in area to the area of a circle of radius 1.
(ii)A regular n – g on is constructible if and only if ϕ (n) is a power of 2.

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