1212-19-505-003

Code No. 0098 / CORE

FACULTY OF SCIENCE

M.Sc. I – Semester Examination, January / February 2020

Sub: Mathematics / Applied Maths / Maths with Computer Science

Paper – I: Abstract Algebra

Time: 3 Hours   Max.Marks: 80

Note: Answer all questions from Part-A and Part-B.
Each question in Part-A carries 4 marks and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
(Short Answer Type)

1 Prove that G acts transitively on X if and only if there is only one orbit.

2 Prove that every finite group has a composition series.

3 Let G be a group of order pq where p and q are primes such that p > q and $q \nmid p$.
Then prove that G is cyclic.

4 Let G be a group. Prove that $\left|\frac{G}{Z(G)}\right| \neq 77$.

5 Let R be a commutative ring with unity. If R has no nontrivial ideals, then prove that R is a field.

6 Prove that $\langle x^4+4\rangle$ is not a prime ideal in **R**[x].

7 Let R be a commutative integral domain with unity in which for each pair
a, b $\in$ R, gcd (a, b) exists. Then prove that if (a, b) = 1 and if a/c and b/c then ab/c.

8 Show that 3 is irreducible but not prime in the ring $\mathbb{Z}[\sqrt{5}]$.

PART – B (4x12 = 48 Marks)
(Essay Answer Type)

9 a) Prove that a group G is nilpotent if and only if G has a normal series.

$$(e) = G_0 \subseteq G_1 \subseteq \ldots \subseteq G_m = G$$

such that $\frac{G_i}{G_{i-1}} \subseteq Z\left(\frac{G}{G_{i-1}}\right)$. i = 1,2, 3, …, m.

OR

b) Let G be a group. If G is solvable, then prove that every subgroup of G and every homomorphic image of G are solvable.

10 a) State and prove Sylow's first theorem.

OR

b) State and prove the fundamental theorem on finitely generated abelian groups.

11 a) If R is a non-zero ring with unity, and I is an ideal in R such that $I \neq R$, then
prove that there is a maximal ideal M of R such that $I \subseteq M$.

OR

b) Let R be a commutative ring and P an ideal in R. Then prove that P is a prtime ideal in
R $\Leftrightarrow$ ab$\in$P, a$\in$R, b$\in$R implies a$\in$P or b$\in$P.

12 a) If R is a UFD, prove that R[x] is also a UFD.

OR

b) If R is a UFD, then prove that the factorization of any element in R as a finite product of irreducible factors is unique to within order and unit factors.

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Code No. 0099 / CORE

FACULTY OF SCIENCE

M.Sc. I – Semester Examination, January / February 2020

Sub: Mathematics / Applied Maths / Maths with Computer Science

Paper – II: Mathematical Analysis

Time: 3 Hours **Max.Marks: 80**

Note: Answer all questions from Part-A and Part-B.
Each question in Part-A carries 4 marks and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
(Short Answer Type)

1 Define a convex set. Prove that balls are convex.

2 Prove that the sum of lengths of open intervals removed while constructing a cantor set is 1.

3 Prove that composition of two continuous functions is continuous.

4 Suppose f is monotonic on (a, b) prove that the set of points of (a, b) at which f is discontinuous is at most countable.

5 Evaluate $\int_0^1 f d\alpha$ where $f = x^3$ and $\alpha = x^4$.

6 If $f \in R(\alpha)$ on (a, b) and if $|f(x)| \leq M$ on [a, b], then prove that

$$\left|\int_a^b f d\alpha\right| \leq M(\alpha(b) - \alpha(a)).$$

7 Discuss with an example that the limit of the integral need not be equal to the integral of the limit even if both are finite.

8 If $f_n \in R(\alpha)$ on [a, b] and if $f(x) = \sum_{n=1}^{\infty} f_n(x)$, $x \in [a, b]$, the series $\sum_{n=1}^{\infty} f_n$ converges uniformly to f on [a,b], then prove that $\int_a^b f d\alpha = \sum_{n=1}^{\infty} \int_b^a f_n d\alpha$.

PART – B (4x12 = 48 Marks)
(Essay Answer Type)

9 a) Let Y be a subset of metric space X. Prove that A subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

OR

b) If P is a non-empty perfect set in IR^k, then prove that P is uncountable.

10 a) Prove that a mapping f of a metric space X into a metric space Y is continuous if and only if $f^{-1}(v)$ is open in X for every open set V in Y.

OR

b) Let f be monotonically increasing on (a, b), then prove that $f(x^+)$ and $f(x^-)$ exist at every point x of (a, b). More precisely $\sup_{a < t < x} f(t) = f(x^-) \leq f(x^+) = \inf_{x < t < b} f(t)$.

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