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Code No. 17555/Core

FACULTY OF SCIENCE
M.Sc. IV - Semester Examination, July 2021
Subject: Mathematics / Applied Mathematics / Mathematics with Computer Science

Paper – I: Integral Equations and Calculus of Variations

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 Convert the initial value problem $y'' + (1 + x^2)y = \cos x,\ y(0) = 0,\ y'(0) = 2$ into an integral equation.

2 Find the resolvent kernel of Volterra integral equation with kernel $K(x,t) = 3^{x-t}$.

3 Show that $\beta(p,q) = \beta(p+1,q) + \beta(p,q+1)$.

4 Solve the integral equation $\phi(x) = \int_{-1}^{1} \frac{xt}{1+\phi^2(t)}\,dt$.

5 State and prove the fundamental lemma of calculus of variations.

6 Distinguish between strong variation and weak variation.

7 Find the extremals of the functional $v[y(x)] = \int_0^1 (1 + y''^2)\,dx$.

8 State and prove Hamilton's principle.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 Solve the Volterra integral equation of the first kind $\int_0^x e^{x-t}\phi(t)\,dt = \sin x$ by reducing it into the Volterra integral equation of the second kind.

10 Explain the method of successive approximations, and use it to solve $\phi(x) = 1 + \int_0^x (x-t)\phi(t)\,dt,\ \phi_0(x) = 1$.

11 Find the characteristic numbers and eigen functions of the integral equation $\phi(x) = \lambda \int_{-1}^{1} (5xt^3 + 4x^2t + 3xt)\phi(t)\,dt$.

12 Solve the boundary value problem $y'' + y = x,\ y(0) = y\left(\pi/2\right) = 0$ using Green's function.

13 Derive the necessary condition for the functional $v[y(x)] = \int_a^b F(x, y, y')\,dx$ with the boundary conditions $y(a) = y_a,\ y(b) = y_b$ to have an extremum.

14 Explain the Brachistochrone problem and find a variational solution to it.

15 Derive the Euler-Poisson equation.

16 State and derive Hamilton's canonical equations.

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Code No. 17556/Core

FACULTY OF SCIENCE
M.Sc. IV - Semester Examination, July 2021

Subject: Mathematics
Paper – II: Elementary Operator Theory

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 Define point spectrum, continuous spectrum, residual spectrum and regular value of a linear operator on a normed space.

2 Prove that eigenvectors $x_1, x_2, \ldots, x_n$ corresponding to different eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of a linear operator T on a vector space X constitute a linearly independent set.

3 Let T be compact linear operator and S a bounded linear operator on a normed space X. Then prove that TS and ST are compact.

4 Prove that every relatively compact subset B of a metric space X is totally bounded.

5 Let T:H→H and W:H→H be bounded linear operators on a complex Hilbert space H and
S = W*TW, then show that if T is self-adjoint and positive, so is S.

6 Prove that the sum of two positive operators is positive.

7 For any projection P on a Hilbert Space H, prove that
$\langle Px,x\rangle = \|Px\|^2$; $P \geq 0$, $\|P\| \leq 1$; $\|P\| = 1$ if $P(H) \neq \{0\}$.

8 Define projection on Hilbert space and spectral family associated with an operator T.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 Prove that all matrices representing a given linear operator T:X→X on a finite dimensional normed space X relative to various bases for X have the same eigenvalues.

10 Show that the spectrum σ(T) of a bounded linear operator T on a complex Banach space X is closed.

11 Prove that (i) every compact linear operator T on a normed space X is bounded
(ii) The identity operator on infinite dimensional normed space X is not compact.

12 Let T:X→X be a compact linear operator on a normed space X and let $\lambda \neq 0$. Then prove that $Tx - \lambda x = y$ ($y \in X$ given) has a solution x if and only if y is such that f(y)=0 for all $f \in X'$ satisfying $T^{\times} f - \lambda f = 0$.

13 Let T be a bounded self-adjoint linear operator on a complex Hilbert space H, $H \neq \{0\}$ and $m = \inf_{\|x\|=1} \langle Tx, x\rangle$, $M = \sup_{\|x\|=1} \langle Tx, x\rangle$, then prove that m and M are spectral values of T.